Int. J. Heat Mass Transfer. Vol. 36, No. 18, pp. 4407–4418, 1993
Printed in Great Britain

0017-9310/93 $6.00+0.00

Empirical correlation equations for heat transfer by forced convection from cylinders embedded in porous media that account for the wall effect and dispersion

R. M. FAND, M. VARAHASAMY and L. S. GREER
Department of Mechanical Engineering, University of Hawaii at Manoa, Honolulu, HI 96822, U.S.A.

(*Received* 17 *February* 1993 *and in final form* 18 *May* 1993)

Abstract—Empirical correlation equations for the Nusselt number have been determined that represent an extensive body of new and previously published experimental data for the geometry under consideration with a level of accuracy that is deemed to be sufficient to render the correlations acceptable for design purposes. The equations cover the Darcy, Forchheimer and turbulent regimes of flow. The empirical correlation equations are based upon a hypothesis that regards the flow in a porous medium to be the superposition of a 'fine' component upon a 'coarse' component and takes into account the wall effect and dispersion.

INTRODUCTION AND STATEMENT OF OBJECTIVE

THIS PAPER deals with heat transfer by forced convection from isothermal horizontal cylinders embedded in infinite liquid-saturated porous media. The matrices of the porous media consist of randomly packed spheres of uniform diameter and the cylinders are subjected to crossflow in the Darcy, Forchheimer and turbulent regimes of flow. Several studies relating to this topic have been published previously, but every such study has dealt with severely limited aspects of the problem; for example, the result of an analytical investigation has been reported, but it is restricted to Darcy flow and uniform porosity, thereby neglecting the so-called 'wall effect'; for another example, the results of an experimental study of the problem have appeared in the literature, but the data base of this study was too narrow in scope to permit an adequate evaluation of the wall effect and the effect of dispersion on the rate of heat transfer.

The objective of the present study was to determine empirical correlation equations for the Nusselt number that adequately account for the wall effect and dispersion for Darcy, Forchheimer and turbulent flow for the geometry considered herein. The achievement of this objective required the acquisition of new experimental data with water as the saturating fluid in an existing apparatus that had been used previously, and it also required the construction and utilization of a new apparatus with which experimental data could be obtained with silicon oil as the saturating fluid.

REVIEW OF THE LITERATURE

Fluid flow in porous media

In order to deal with the problem of forced convection heat transfer from a horizontal cylinder embedded in a porous medium in the presence of cross-flow, it is first necessary to have certain information concerning the (isothermal) flow of fluids through porous media. Fand *et al.* [1] have studied the three recognized regimes of flow through infinite porous media, namely, the Darcy regime (where $Re_d \leqslant Re_{DH} = 2.3$),† the Forchheimer regime (where $5 = Re_{FL} \leqslant Re_d \leqslant Re_{FH} = 80$) and the turbulent regime (where $Re_d \geqslant Re_{TL} = 120$) plus the two regions of transition between these three regimes. The values of the lower and upper bounds of the flow regimes have been established using experimental data obtained for dimension ratios $D_c/d \geqslant 1.4$, where D_c represents the diameter of a cylinder used to confine the matrix of a porous medium and d is the diameter of solid spherical particles constituting the matrix.

In the Darcy regime, where inertial forces are negligible in comparison with viscous forces, the volume rate of flow is proportional to the negative of the pressure gradient and the following equation applies:

$$\frac{P'd}{\mu u} = \frac{d}{K} \tag{1}$$

where P' represents the negative of the pressure gradient in the direction of flow, u is the volumetric rate of flow

† The subscript DH is a mnemonic device which refers to the highest value (H) of the particle Reynolds number for which Darcy (D) flow occurs. Similar subscripts will be used to indicate the lowest value (L) of the particle Reynolds number for which a particular type of flow occurs.

NOMENCLATURE

A, A' first Ergun constants per equations (4) and (5)
A_m mean flow area per equation (31)
A_c cross sectional area of cylinder (test section) used to contain porous media
A_w, A'_w first Ergun–Reichelt parameters per equations (14) and (15)
B, B' second Ergun constants per equations (7) and (8)
B_w, B'_w second Ergun–Reichelt parameters per equations (14) and (15)
C_{F1}, C_{F2} Forchheimer coefficients per equation (4)
c_{pf} specific heat of a (saturating) fluid at constant pressure
c_s specific heat of the solid particles in a porous medium
C_{T1}, C_{T2} turbulent coefficients per equation (5)
D diameter of a heated (test) cylinder
d diameter of a spherical particle
D_c diameter of a cylinder (test section) used to contain porous media
Di dimensionless measure of dispersion per equation (36)
e base for natural logarithms
f' modified friction factor, $P'd/\rho\mu\beta_\varepsilon$
f_w wall modified friction factor, f'/M
g gravitational constant; also denotes a functional relationship
Gr_D Grashof number, $gD^3\beta\Delta T/\nu^2$
Gr_K Grashof number, $gKD\beta\Delta T/\nu^2$
h heat transfer coefficient
K permeability per equation (2)
k generic symbol for thermal conductivity
k_e effective thermal conductivity per equation (19)
k_{ew} wall corrected effective thermal conductivity per equation (21)
k_f thermal conductivity of the fluid saturating a porous medium
k_s thermal conductivity of the solid particles in a porous medium
k_λ effective thermal conductivity per equation (22)
$k_{\lambda w}$ wall corrected effective thermal conductivity per equation (23)
L_c length of a cylinder (test section) whose diameter is D_c
M wall correction factor per equation (10)
N number of data
Nu Nusselt number, hD/k
P' negative of the pressure gradient in the direction of flow, $-\Delta P/L_c$
ΔP difference between the outlet and inlet pressures of a cylinder (test section) containing a porous medium through which a fluid is flowing
Pr Prandtl number, $\mu c_{pf}/k$
Pr_e effective Prandtl number per equation (28)
R_H hydraulic radius, $\varepsilon d/6(1-\varepsilon)M$
Re_D cylinder Reynolds number, $Re_D = uD/\nu$
Re_d particle Reynolds number, $Re_d = ud/\nu$
Re'_d modified particle Reynolds number, $Re'_d = Re_d/(1-\varepsilon)$
Re_w wall modified Reynolds number, $Re_w = Re'_d/M$
s_0 specific surface area (surface area per unit volume of the particles composing a porous medium: for spheres, $s_0 = 6/d$)
T temperature
T_b bulk temperature; refers to the temperature in a fluid outside of a thermal boundary layer
T_{cyl} refers to the temperature in a fluid at the surface of a heated (test) cylinder
ΔT temperature difference, $T_{cyl} - T_b$
u flow velocity in a porous medium
u_c test section core flow velocity
u_m measured average test section flow velocity.

Greek symbols
α thermal diffusivity, $k_f/\rho c_{pf}$
α_e effective thermal diffusivity of a porous medium per equation (29)
α_ε a function of ε, $(1-\varepsilon)^2/\varepsilon^3$
β coefficient of volumetric expansion of a fluid
β_ε a function of ε, $(1-\varepsilon)/\varepsilon^3$
ε porosity
ε_ω wall corrected porosity, $\varepsilon[1+0.5(d/D)^3]$
κ Kozeny–Carman factor
λ conductivity ratio, k_f/k_s
μ dynamic viscosity of a fluid
ν kinematic viscosity of a fluid, μ/ρ
ρ density of a fluid
ρ_s density of the solid particles in a porous medium
$(\rho c)_e$ effective heat capacity of a porous medium, $\varepsilon\rho c_{pf} + (1-\varepsilon)\rho_s c_s$.

Subscripts
cal refers to a calculated value
exp refers to an experimentally determined value.

Error notation
E percent error: $[(Nu_{exp} - Nu_{cal})/Nu_{exp}] \times 100\%$
E_{md} percent mean deviation of error, $\Sigma_{i=1}^{N} |E_i|/N$.

per unit area (sometimes referred to as the 'Darcian speed' or the 'velocity' when the context makes the direction clear), μ is the dynamic viscosity of the fluid, and K is a constant of proportionality called the permeability. The following semi-empirical equation for K accurately represents many experimental data:

$$K = (\kappa s_0^2 \alpha_\varepsilon)^{-1}; \quad \alpha_\varepsilon = \frac{(1-\varepsilon)^2}{\varepsilon^3}, \tag{2}$$

where ε is the porosity defined as the void fraction of the total volume of the porous medium, s_0 is the specific surface area of the particles composing the porous medium, and κ is a constant, called the Kozeny–Carman factor. For a porous medium composed of spheres of uniform diameter d, $s_0 = 6/d$ so that

$$K = \frac{d^2}{36\alpha_\varepsilon \kappa}. \tag{3}$$

In the Forchheimer regime, inertial effects become significant and the following equation applies:

$$\frac{P'd}{\mu u} = C_{F1}d + C_{F2}\, Re_d, \tag{4}$$

where $C_{F1}d = A\alpha_\varepsilon/d$ and $C_{F2} = B\beta_\varepsilon/d$ with $\beta_\varepsilon = (1-\varepsilon)/\varepsilon^3$. A and B are non-dimensional constants called the first and second Ergun constants for Forchheimer flow, respectively.

It has been found that equation (4) applies to turbulent flow if the constants therein are replaced as follows:

$$\frac{P'd}{\mu u} = C_{T1}d + C_{T2}\, Re_d, \tag{5}$$

where $C_{T1}d = A'\alpha\varepsilon/d$ and $C_{T2} = B'\beta_\varepsilon/d$.

It will be useful to note that equations (1), (4) and (5) may be rewritten, respectively, as follows:

$$f' = \frac{36\kappa}{Re_d'}, \tag{6}$$

$$f' = \frac{A}{Re_d'} + B, \tag{7}$$

$$f' = \frac{A'}{Re_d'} + B', \tag{8}$$

where $f' = P'd/\rho u^2 \beta_\varepsilon$ and $Re_d' = Re_d/(1-\varepsilon)$ are called the modified friction factor and the modified particle Reynolds number, respectively. The values of the constants in equations (6)–(8) were determined experimentally in ref. [1] to be $\kappa = 5.34$, $A = 182$, $B = 1.92$, $A' = 225$ and $B' = 1.61$.

The transition regions between the Darcy and Forchheimer flow are difficult to characterize, because they cannot be represented by simple equations such as equation (6), (7) or (8). It was shown in ref. [1] that this difficulty can be overcome without incurring excessive error by assuming that fictitious 'transition Re_d's' exist, denoted by Re_{DF} and Re_{FT} ($Re_{DF} = 3$, $Re_{FT} = 100$), at which the flow abruptly changes from Darcy to Forchheimer and from Forchheimer to turbulent flow.

The effect of the wall on flow in a porous medium

The preceding equations demonstrate that the porosity, ε, is a primary controlling geometrical parameter. Now, when a porous medium whose matrix is composed of discrete solid particles is confined in a duct, the wall of the duct affects the local magnitude of the porosity, because the spatial distribution of the particles must conform with the shape of the wall. This is called the 'wall effect'. For the case of spherical particles confined in a circular cylinder, to which the present study is restricted, the porosity tends toward unity upon approach to the cylinder wall. Further, Roblee *et al.* [2] have observed that the local porosity near a confining cylindrical wall varies cyclically in a zone extending to three particle diameters from the wall into a bed of spheres of uniform diameter. Benenati and Brosilow [3] have reported that the zone, within which oscillation in local porosity occurs, extends inward from a cylinder wall a distance of approximately five spherical diameters. Within this annular 'zone of the wall', the average porosity is greater than it is without, and hence, in the presence of a uniform pressure gradient, the averge velocity of the flow is higher within the zone than without. This effect is commonly referred to as 'channeling'. Clearly, the annular zone of the wall comprises an increasing fraction of the cross-sectional area of the cylinder as the dimension ration D_c/d decreases. Therefore, the influence of the wall upon the flow (via channeling) becomes more significant as D_c/d progressively decreases.

Many investigators have studied the effect of the wall. Those particularly relevant to this study are Mehta and Hawley [4], Reichelt [5], and Fand and Thinakaran [6]. In ref. [4] a hydraulic radius, R_H, was defined:

$$R_H = \frac{\varepsilon d}{6(1-\varepsilon)M}, \tag{9}$$

where

$$M = 1 + \frac{2}{3}\left[\frac{d}{(1-\varepsilon)D_c}\right]. \tag{10}$$

Based on this definition of M, together with the 'wall modified' parameters, defined by

$$f_w = \frac{f'}{M} \tag{11}$$

and

$$Re_w = \frac{Re_d'}{M}, \tag{12}$$

equations (6)–(8) can be written as follows:

$$f' = \frac{36\kappa_w}{Re_d'}, \tag{13}$$

$$f_w\ Re_w = A_w + B_w\ Re_w, \tag{14}$$

$$f'_w\ Re_w = A'_w + B'_w\ Re_w. \tag{15}$$

The quantity κ_w is the wall-corrected Kozeny–Carman factor. A_w, B_w and A'_w, B'_w are referred to as the first and second Ergun–Reichelt parameters for Forchheimer and turbulent flow, respectively.

Reichelt [5] determined numerical values of A_w and B_w that have been superseded by more recent information contained in ref. [6]. It was shown in ref. [6] that each of the five wall-corrected flow parameters can be represented by correlation equations having the following common form:

$$Y_w = Y - a\,e^{-f(D/d)}, \tag{16}$$

where Y_w represents a wall-corrected parameter, $f(D/d) = p(D/d)^3 + q(D/d)^2 + r(D/d)$ and a, p, q and r are numerical correlation constants. The values of the correlation constants for all five flow parameters are listed in Table 1.

Heat transfer: empirical results

When a heated horizontal cylinder is subjected to a cross-flow of fluid, natural and forced convection effects always occur simultaneously. Therefore, for purposes of the present study of *forced convection*, it is necessary to have a criterion whereby the relative importance of natural convection effects can be judged. Fand and Keswani [7] have published results of an experimental investigation of combined natural and forced convection heat transfer from a horizontal cylinder to water with cross-flow which shows that when

$$\frac{Gr_D}{Re_D^2} \leqslant 0.5, \tag{17}$$

the predominant heat transfer mechanism is forced convection. Although the investigation in ref. [7] did not involve porous media, Fand and Phan [8] later found empirical evidence that equation (17) is applicable to a cylinder embedded in a porous medium if Gr_D is replaced by Gr_K. It was also determined in ref. [7] that the following correlation applies to the case of a heated horizontal cylinder in a cross-flow of water (no porous medium) when forced convection is predominant:

$$Nu = (0.255 + 0.699 Re_D^{0.5}) Pr^{0.29}, \tag{18}$$

where all of the fluid properties are evaluated at the mean film temperature. Equation (18) was used in ref. [8] to formulate a correlation hypothesis for forced convection heat transfer for a similar configuration within a porous medium.

Fand et al. [9] have suggested that the influence of the wall effect on the rate of heat transfer from an embedded cylinder can be taken into account by replacing ε where it appears in the following definition of the effective thermal conductivity, k_e, of the porous medium,

$$k_e = \varepsilon k_f + (1-\varepsilon)k_s, \tag{19}$$

by the so called 'wall corrected porosity', ε_w defined as:

$$\varepsilon_w = \varepsilon\left[1 + \frac{1}{2}\left(\frac{d}{D}\right)^3\right], \tag{20}$$

where D is the diameter of the embedded cylinder. The wall corrected effective thermal conductivity is then:

$$k_{ew} = \varepsilon_w k_f + (1-\varepsilon_w)k_s. \tag{21}$$

In equations (19) and (21), k_f and k_s are the thermal conductivities of the saturating fluid and the solid particles in the porous matrix, respectively.

Equation (19), along with other correlations for the effective thermal conductivity of porous beds of spheres saturated with liquids, have been tested by Prasad et al. [10] against experimental data. This work demonstrates that the accuracy of equation (19) decreases as the ratio of the thermal conductivities, $\lambda = k_f/k_s$, deviates from unity. An expression that better represents experimental data in ref. [10] for porous media with $\lambda = 0.14$ (as with silicon oil and glass) and $\lambda = 0.59$ (as with water and glass) is

$$k_\lambda = k_f \lambda^{-n}, \tag{22}$$

where $n = 0.280 - 0.757 \log_{10} \varepsilon + 0.057 \log_{10} \lambda$. If ε in equation (22) is replaced by ε_w, in order to account for the influence of the wall effect on conductivity, a 'wall-corrected k_λ' can be defined as follows:

$$k_{\lambda w} = k_f \lambda^{-nw}, \tag{23}$$

where $nw = 0.280 - 0.757 \log_{10} \varepsilon_w + 0.057 \log_{10} \lambda$.

Correlation equations applicable to the case of heat transfer from an embedded horizontal cylinder in cross-flow of water through a porous medium for a single value of D/d ($D/d = 3.73$) are presented in ref. [8]. These correlations were generated using a hypothesis consisting of four assumptions, the pri-

Table 1. Numerical values of constants in equation (16)

Y_w	Y	a	p	q	r
κ_w	5.340	0.6545	0	0	0.09034
A_w	172.9	82.18	0.0001125	−0.003931	0.1314
B_w	1.871	1.636	0.0004908	−0.01665	0.2925
A'_w	213.7	129.7	0.00003852	−0.003376	0.1510
B'_w	1.569	1.350	0.0003688	−0.01465	0.2646

mary assumption being that flow in this situation can be regarded as consisting of two components: a 'coarse' flow having streamlines analogous to those near a heated cylinder immersed in a fluid without a porous medium present; and a superimposed 'fine' flow that represents the meandering motion of the fluid through the interstitial spaces in the porous medium. The remaining three assumptions provide the rationale for stating that heat transfer from a heated cylinder in a porous medium in cross-flow can be described by an equation similar to equation (18), but multiplied by functions containing powers of the Prandtl and Reynolds numbers. Using such reasoning, it was determined that the following equations adequately represent forced convection heat transfer from a horizontal cylinder embedded in a porous medium with cross-flow of water for $D/d = 3.73$:

$$Nu = 2.17(0.255+0.699\,Re_D^{0.5})Pr^{0.188}\,Re_d^{0.230}; \quad k = k_{ew} \quad (24)$$

for $0.5 < Re_d \leqslant 3$;

$$Nu = 2.15(0.255+0.699\,Re_D^{0.5})Pr^{0.154}\,Re_d^{0.126}; \quad k = k_{ew} \quad (25)$$

for $3 < Re_d \leqslant 100$;

$$Nu = 1.48(0.255+0.699\,Re_D^{0.5})Pr^{0.290}\,Re_d^{0.179}; \quad k = k_{ew} \quad (26)$$

for $Re_d > 100$.

The thermophysical properties contained in all correlation equations considered in this paper are evaluated at the mean film temperature unless specified otherwise.

Heat transfer: analytical results

Cheng [11] has derived an expression for the local heat flux distribution around a heated cylinder embedded in a porous medium for Darcy flow by using boundary layer approximations to obtain a similarity solution to the mathematical equations governing the conservation of mass, momentum, and energy in the fluid about the cylinder. Nield and Bejan [12] integrated this expression to obtain a result for the average Nusselt number that is applicable to the geometry of the present investigation:

$$Nu = 1.015(Re_d\,Pr_e)^{0.5}, \quad (27)$$

where Pr_e is an effective Prandtl number of the porous medium defined as:

$$Pr_e = \frac{\nu}{\alpha_e}. \quad (28)$$

Here the effective thermal diffusivity of a porous medium α_e is defined as

$$\alpha_e = \frac{k_e}{(\rho c)_e} \quad (29)$$

with

$$(\rho c)_e = \varepsilon\rho c_{pf}+(1-\varepsilon)\rho_s c_s. \quad (30)$$

In equation (30), ρ_s and c_s are the density and the specific heat of the solid particles in the porous medium, respectively; and ρ and c_{pf} are, respectively, the density and the specific heat at constant pressure of the saturating fluid. The Nusselt number in equation (27) is calculated using k_e.

Dispersion

A phenomenon that occurs in all flows through porous media (whether or not there is heat transfer), and therefore must be considered in this study, is 'dispersion'. The meaning of the term dispersion can be explained qualitatively by comparing the one-dimensional laminar flow of a fluid through a region of space in the presence of, and in the absence of, a porous matrix. In the absence of a porous matrix, the path of all fluid particles are straight, parallel lines; whereas, in the presence of a porous matrix, each fluid particle follows a tortuous path through the interstices of the porous medium. The trajectory of each fluid particle in a porous medium is a random process, the result of which is an overall transverse migration, or 'dispersion', of the particles away from the straight, parallel lines they would have followed in the absence of the porous matrix. Dispersion affects the transfer of heat because it causes mixing due to the aforementioned transverse migration.

Dispersion is a complex phenomenon. A description of it as a second order tensor is provided in ref. [12] in the presence of heat transfer. The components of this tensor are, for a given geometry (characterized by d in the present case), functions of the Reynolds number of the flow, the effective thermal diffusivity, the magnitude of velocity of the flow through the interstitial spaces in the porous medium and the pore size of the porous medium (which, in porous media consisting of spheres of uniform diameter is, in turn, a function of the particle diameter d).

EXPERIMENTAL APPARATUS, PROCEDURE AND DATA

Two different apparatuses were used to gather experimental data in this investigation. The first of these is a high-precision stainless steel water tunnel which had been previously used to obtain the data reported in ref. [8] and is described therein. This water tunnel is equipped with a set of calibrated orifices that permit measurement of the volume rate of flow through the tunnel. The test section of the water tunnel, into which heated cylinders and porous media were inserted, was removable and could be incorporated into the second apparatus, which consisted of a loop through which silicon oil (20Cs) could be pumped at various measured volume rates of flow; thus, the second apparatus comprised an 'oil tunnel' whose test section was identical to that of the aforementioned water tunnel. The diameter and length of the test section were 0.08660 m and 0.4572 m, respectively. The test section contained a calibrated copper–constantan thermocouple to measure the upstream (bulk) temperature of the flowing fluid (water or oil, as the case may be). Limitations in pumping capacity restricted the oil tunnel to Darcy and Forchheimer flow.

Two isothermal electrically heated cylinders having different diameters, D, were inserted transversely into the test section. The test section was packed with porous media whose matrices consisted of uniform soda-lime glass spheres having different diameters, d. The cylinders were maintained at various temperatures by controlling the (measured) electric current flowing through them while subjecting them to various (measured) fluid velocities.

The heated cylinders consisted of thin-walled stainless steel tubes, or 'shells', which contained three contiguous close-fitting electrically insulated copper inserts as shown in Fig. 1. The cylinders were heated by passing direct electric current (order of 100 amperes) through the stainless steel shells. The copper inserts served to equalize the temperature of the inner surfaces of the shells, and this inner surface temperature was indicated by a calibrated thermocouple located centrally within the heated specimens as indicated in Fig. 1. The outer surface temperature of the heated specimens, which is needed to calculate the heat transfer coefficient, was obtained from the internal thermocouple reading by solving the controlling different equation for conduction in a radial system with uniform heat generation. The inner surfaces of the shells were assumed to be adiabatic for purposes of these calculations. It was found that the differences in temperature between the inner and outer surfaces of the thin shells were relatively small and in most cases negligible. Uninsulated copper rods having very low electrical resistance, which functioned as electric terminals, were inserted into the shells at both ends (see Fig. 1) to a depth which insured that electrical heating was confined to only these portions of the shells that were in contact with the porous media.

This design renders negligible the axial conduction of heat away from the middle of the test specimen, so that the measured rate of heat generation in its middle portion equals the rate of heat transfer by convection at this locus. This experimentally determined rate of convection, together with simultaneous determinations of the bulk fluid and specimen surface temperatures, provided the means to calculate the heat transfer coefficient at the middle of the specimen. The middle of each specimen was in all cases more than five particle diameters away from the confining wall of the test section, thereby avoiding the 'zone of the wall' associated with the test section.

The data from ten series of experimental tests were obtained and considered herein. The ranges of the experimental parameters for these ten series are listed in Table 2. Each test series was designated by the letters TS followed by a letter that indicates the fluid medium (W for water or O for oil), followed by the specific test series member (from 1 to 10). The data of each test series were divided into subsets, depending upon whether they fell into the Darcy, Forchheimer, turbulent or transition ranges of Re_d. These subsets were designated by adding the letters D (for Darcy), F (for Forchheimer), T (for turbulent), DF (for transition from D to F) and FT (for transition from F to T) to the parent test series symbol; thus, TSW3D refers to the subset of TSW3 that contains data per-

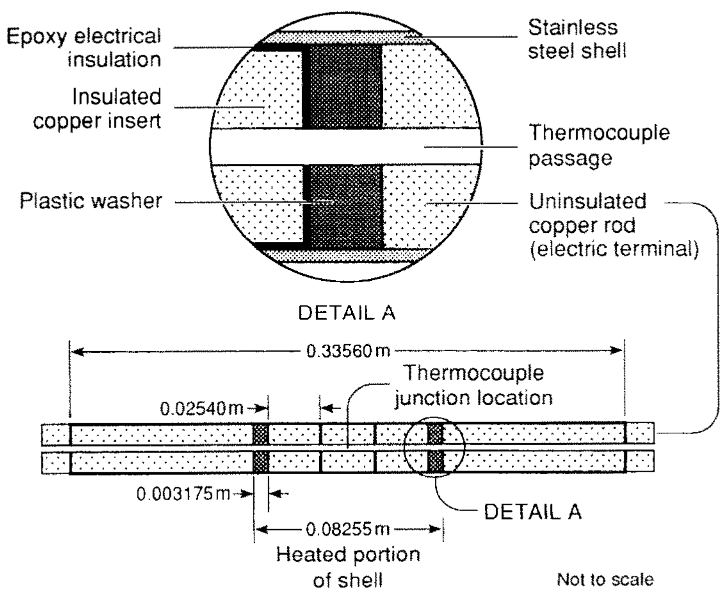

FIG. 1. Drawing of test cylinder.

Table 2. Ranges of experimental parameters

Test series	No. of data (N)	D (mm)	d (mm)	D/d	Range of ΔT (°C)	Range of Re_d	Range of Pr ($k = k_{\lambda w}$)
TSW1	57	11.450	2.098	5.458	2.9–5.2	0.23–210	4.5–4.9
TSW2	95	11.450	3.072	3.727	5.6–41.6	0.64–250	2.9–4.9
TSW3	47	8.509	3.038	2.801	5.1–5.2	0.46–210	4.6–4.7
TSW4	46	11.450	4.992	2.293	5.1–5.2	0.91–370	4.8–5.1
TSW5	48	8.509	4.029	2.112	5.1–5.2	0.82–290	4.7–4.8
TSW6	47	11.450	5.969	1.918	5.1–5.2	1.3–450	4.8–5.1
TSW7	46	8.509	4.992	1.704	5.1–5.2	1.5–390	4.4–4.6
TSW8	45	8.509	5.969	1.426	5.1–5.2	1.8–430	4.9–5.3
TSO9	68	11.450	3.038	3.769	6.5–42.3	1.4–24	38–54
TSO10	66	8.509	3.038	2.801	2.9–46.6	1.1–25	34–56

taining to Darcy flow. All data considered here were obtained in the course of the present investigation except for TSW2T, which was abstracted from the data obtained by Fand and Phan [8]. The reason for incorporating TSW2T into the present study was that TSW2T covers a wider range of ΔT for turbulent flow (and hence a wider range of Pr) than was obtainable with the present apparatus.

As stated in the Introduction, this study purports to deal with heat transfer by forced convection from cylinders embedded in *infinite* porous media. The adjective infinite here implies media so large in extent that wall effects associated with their confining surfaces do not appreciably affect the convection process. It was shown by Fand and Thinakaran [6] that if a porous medium whose matrix is composed of uniform spheres is confined within a circular cylinder, the cylinder diameter must be at least forty times the sphere diameter in order to render wall effects negligible. It was not possible, for practical reasons, to acquire a test section sufficiently large as to satisfy the condition $D_c/d \geqslant 40$ for use in the present study. The highest achievable value was $D_c/d = 14.5$.† Therefore, in order to interpret the experimental data obtained here in terms of infinite porous media, it was necessary to calculate the velocity in the central portion or 'core' of the test section where the porosity, ε, is uninfluenced by the confining wall and is identical to the porosity in an infinite randomly packed porous medium: $\varepsilon = 0.360$ for spherical particles. This core velocity, designated by u_c, was calculated for each experiment by a method that is described in what follows. The core velocity was then corrected for blockage caused by the presence of the heated test specimens in the field of flow in order to obtain the final value of the velocity, u, used to correlate the heat transfer data.

Experimentally measured volume rates of flow provided information whereby the mean velocities, u_m, in the test section could be calculated. Corresponding to each of these mean velocities, equations (13)–(15) were employed to obtain the corresponding wall modified friction factors, and thereby the pressure gradients in the test section. The calculated values of the pressure gradients, which are assumed to be uniform across the cross-sectional area of the test section, A_c, were then inserted into equations (6)–(8) to calculate the flow velocity, u_c, in the core of the test section, where the porosity is not affected by the wall effect.

The method used to account for blockage is identical to that used in refs. [8, 13], and is described in Vliet and Leppert [14]. With this method, a mean flow area, A_m, defined as the ratio of the net flow volume at the location of the test cylinder to the diameter of the test cylinder, is expressed as follows:

$$A_m = \frac{\dfrac{\pi(D_c^2)D}{4} - \dfrac{\pi(D^2)D_c}{4}}{D}. \tag{31}$$

The velocity, u, of the fluid corrected for flow blockage is then computed from the continuity equations for incompressible flow to be:

$$u = \frac{A_c u_c}{A_m}. \tag{32}$$

The calculation of u_c based upon u_m is considered to be precise, but the blockage correction is approximate for several reasons, primary among which is that it does not account for the wall effect associated with the surface of the heated cylinder.

It is estimated that the experimental errors in the determination of the Nusselt and Reynolds numbers do not exceed 5% for any single set of measurements included in Table 2.

EVALUATION OF PREVIOUSLY PUBLISHED CORRELATION EQUATIONS

In this study, four previously published correlation equations were tested against relevant experimental data listed in Table 2. This was done by computing the maximum error and the mean deviation of the Nusselt number (see Error Notation in Nomenclature) within each of the relevant data subsets. In order to decide whether a particular correlation equation is satisfactory, it was necessary to adopt an appropriate criterion. In view of the magnitudes of the exper-

† This figure refers to the 'worst case', for which $D_c = 0.08660$ m and $d = 0.00600$ m.

imental errors discussed above, plus the fact that the packing of the test section with the porous media was random, the following arbitrary criterion, referred to hereafter as the 'criterion of acceptability', was adopted: a correlation equation that exhibits mean deviations of error with respect to each relevant data subset not exceeding 10%, and a maximum error for any single relevant datum not exceeding 20% is considered to be acceptable. This criterion has been adopted in the belief that correlations that satisfy it are sufficiently accurate to be used for purposes of design.

The four previously published correlations that were evaluated for accuracy are equations (24) and (27) for Darcy flow, equation (25) for Forchheimer flow, and equation (26) for turbulent flow. The evaluations were initially performed with the Nusselt and Prandtl numbers calculated precisely in accordance with the definitions of properties adopted by the authors of these equations. Since the initial evaluations of these four equations yielded errors *vis-à-vis* the data that were deemed to be excessive, and in an effort to reconcile these equations with the data, they were re-evaluated by substituting $k_{\lambda w}$ in place of the effective thermal conductivities defined by the equations' authors. This substitution was suggested by the reported superiority of k_λ in representing thermal conductivity, and by the reported success in accounting for the effect of the wall on the rate of heat transfer from an embedded cylinder achieved by replacing ε with ε_w. With this modification, the errors incurred by the previously published equations were still found to be excessive. A third and final effort to reconcile the published equations with the data was made, this time by replacing the effective Prandtl number, Pr_e, in equation (27) by Pr as defined in equations (24) and (25). This final effort resulted in a marked improvement but still yielded excessive errors.† It was concluded that none of the previously published equations represents the data with sufficient accuracy, because none of these equations adequately accounts for the effect of the wall and of dispersion. Consequently, a new hypothesis that purportedly accounts for the effect of the wall and dispersion was formulated, as is discussed below. This new hypothesis led to the determination of correlation equations that represent the experimental data with a degree of accuracy consistent with the criterion of acceptability.

A NEW CORRELATION HYPOTHESIS

An empirical correlation of experimental data is usually developed by adopting an appropriate hypothesis which consists of a mathematical equation containing arbitrary constants, and then determining the numerical values of the constants by fitting the equation to the experimental data. Since no theoretical solution to the present problem is available, the method followed here to determine a new correlation hypothesis was to adopt mathematical forms based on previous relevant experience, and then modify these forms based upon physical reasoning not heretofore applied.

In order to correlate the experimental data assembled in this study, it was assumed that the Nusselt number is expressible as the product of two functions, f_1 and f_2, the first of which represents the influence of the coarse flow on heat transfer and the second that of the fine. This assumption can be expressed symbolically as follows:

$$Nu = f_1 f_2. \tag{33}$$

The function f_1 was taken to be a generalization of equation (27), namely,

$$f_1 = C_1\, Re_d^{0.5}\, Pr^a \tag{34}$$

where C_1 and a are constants. The function f_2 was taken to be

$$f_2 = C_2\, Pr^b g(Di, D/d), \tag{35}$$

where C_2 and b are constants and Di is a dimensionless measure of the dispersion in the free stream. The function g is a multiplying factor that represents the effect of dispersion on the Nusselt number via its influence upon Pr. The dimension ratio D/d in g accounts for the interaction between the fine and coarse flows.‡ The need to include D/d as an independent variable in g becomes clear when one considers the fact that dispersion in the free stream in a porous medium with given d is independent of the diameter D of a heated cylinder embedded therein, and hence such dispersion cannot affect the Nusselt number equally for all D; this implies that the scale ratio D/d must be included as an independent variable in the function g. The reasoning that led to a specific choice of the form of the function g will now be explained.

The first step in structuring g was to determine a suitable measure of dispersion, Di. Now, Di was known to be a function of Re_d; however, because of the complexity of the process, it was anticipated that the determination of an appropriate measure of dispersion that is expressible directly as a function of Re_d and also possesses certain other requisite properties (to be described presently) would be difficult to achieve. It was surmised that a suitable measure of dispersion could be more readily determined in terms of the *pressure gradient* in the free stream, via the dimensionless friction factor f', which is, in turn, a function of Re_d per equations (6)–(8). However, f', by itself, does not represent a suitable choice for Di for the following reason: consider a given heated cylinder (given D) maintained at a given surface temperature

† The mean deviation with respect to at least one of the relevant data subsets is greater than 25% in every evaluation of equations (24)–(27) described herein.

‡ Note that $D/d = Re_D/Re_d$ for constant density and viscosity. Re_D/Re_d may be regarded as the ratio of the scale of the coarse flow to that of the fine.

Table 3. Numerical values of constants in equation (37)

Flow regime	C	f	d	c	e
Darcy	1.248	0.3534	0.05355	0.5	0.5467
Forch.	0.6647	0.2286	0.2090	0.5	1.417
Turb.	0.7956	0.06036	0.2248	0.5	1.588

and suppose that the cylinder is successively embedded in a series of porous media that are saturated by the same fluid and are subject to the same crossflow velocity (identical Re_D) but have diminishing particle sizes (d and $Re_d \to 0$). For such a series of experiments one should expect the Nusselt number for the cylinder to approach a finite limit. But f' increases beyond all bounds as $Re_d \to 0$, and hence f' is not a suitable measure of dispersion in the function g. However, the quantity $f'Re'_d$ is suitable from this point of view, because this quantity remains finite as $Re_d \to 0$. Hence, the measure of dispersion adopted here is:

$$Di = f' \; Re'_d. \tag{36}$$

The next step in structuring g was to determine a form involving D/d that would account for the interaction between the fine and coarse flows. Our first impulse was to adopt a simple function, such as $(D/d)^c$; however, upon reflection, this simple function was discarded for the same reason that f' was dis-

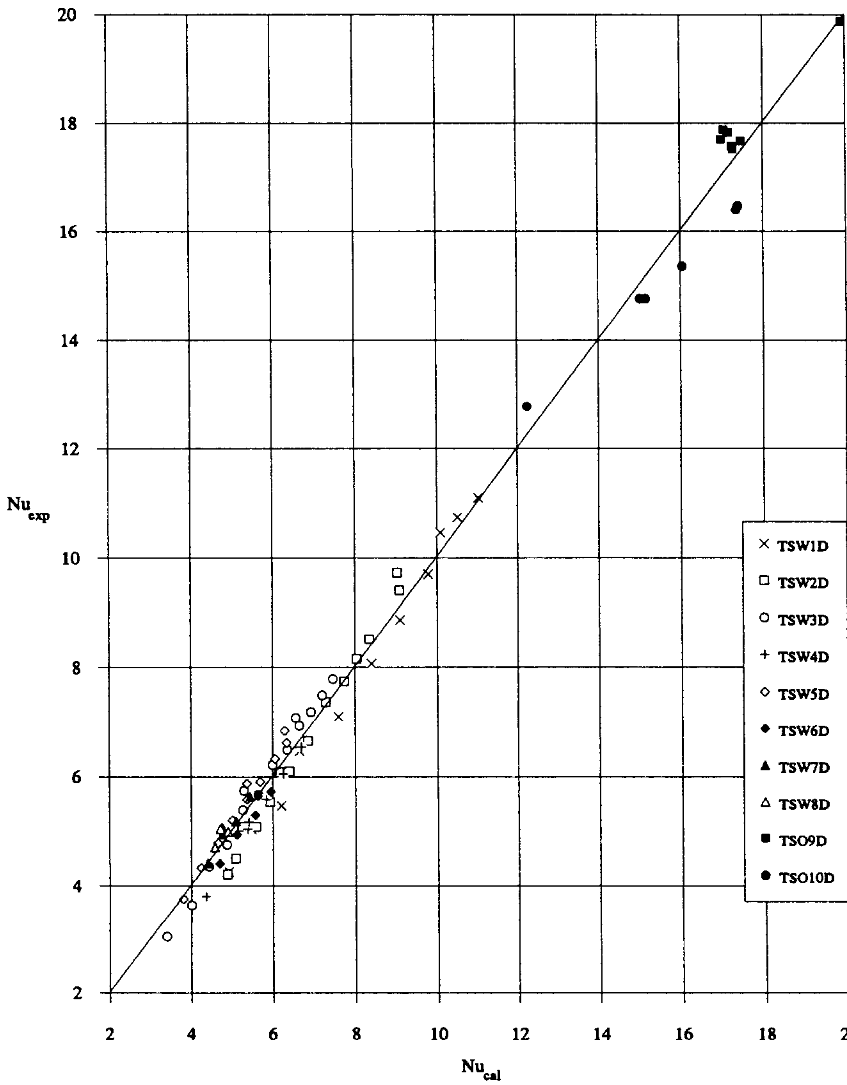

FIG. 2. Graph of equation (37) for Darcy flow.

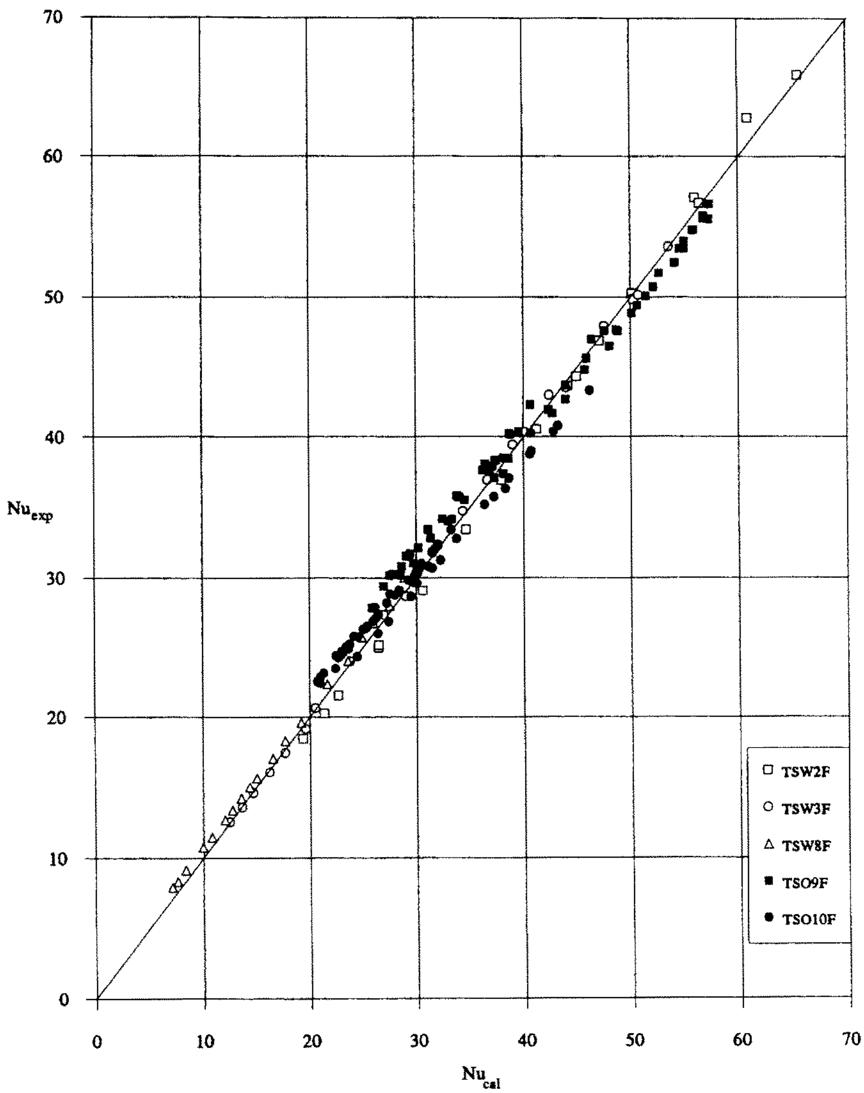

FIG. 3. Graph of equation (37) for Forchheimer flow.

carded as a measure of Di; more explicitly, the function $(D/d)^c$ exceeds all bounds as $d \to 0$. A power function of arctan $(D/d)^c$ was finally adopted because arctan $(D/d)^c$ monotonically approaches a finite upper bound ($\pi/2$) as $d \to 0$. The foregoing reasoning led to the adoption of the following form for g:

$$g = C_2(f'\ Re'_d)^d\,[\arctan\,(D/d)^c]^e$$

where C_2, c, d and e are constants. Taken together, the preceding reasoning and assumptions lead to the following correlation hypothesis for Nu:

$$Nu = C\,Re_D^{0.5}\,Pr^f(f'\ Re'_d)^d\,[\arctan\,(D/d)^c]^e;\quad k = k_{\lambda w}, \qquad (37)$$

where $C = C_1C_2$ and $f = a+b$.

EVALUATION OF THE NEW CORRELATION HYPOTHESIS

The optimum values of the constants in equation (37) were obtained by first choosing a value for c and then applying the method of nonlinear regression to the equation with respect to the relevant data identified in Table 2.† The density and viscosity in Re'_d were evaluated at the bulk temperature and all other properties were evaluated at the mean film temperature. The optimum numerical values of the constants so obtained for Darcy, Forchheimer and turbulent flow are listed in Table 3. With these constants, equa-

† More precisely, a series of values of c was explored and the value of c that produced the optimum final result was adopted. The initial values adopted for the nonlinear regression were obtained by applying linear regression to the logarithm of equation (37).

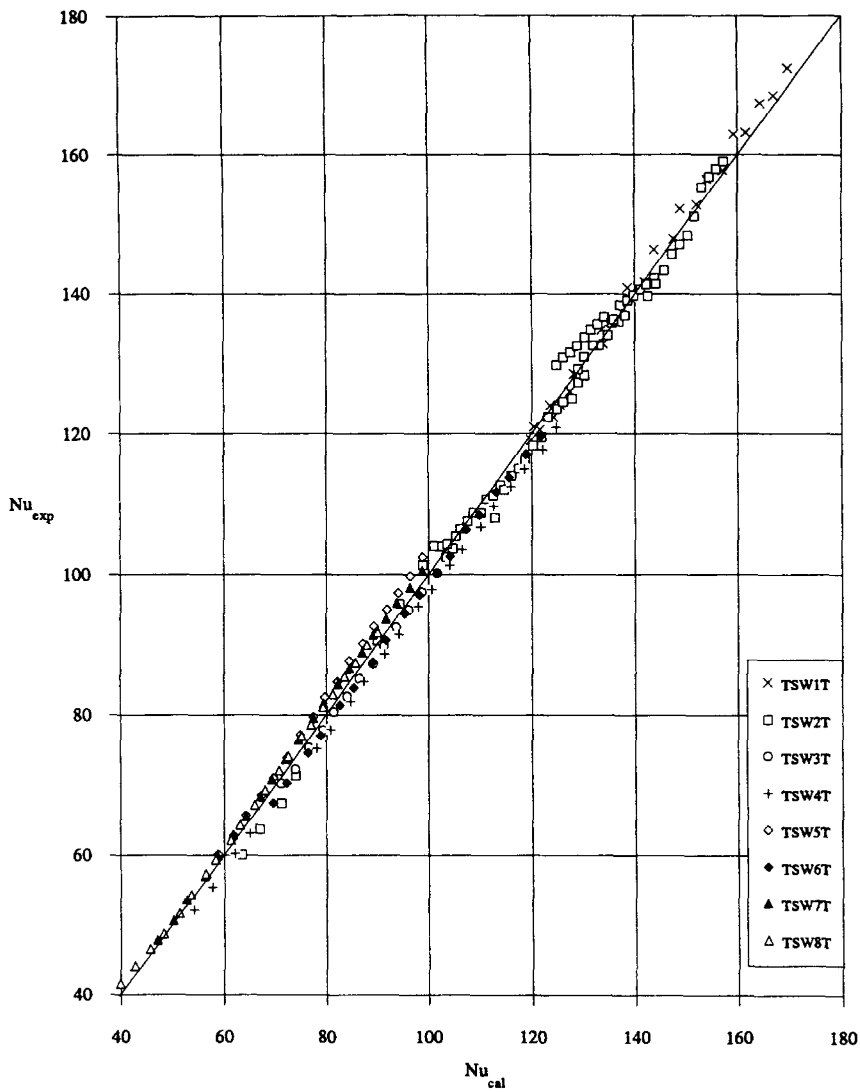

FIG. 4. Graph of equation (37) for turbulent flow.

tion (37) behaves with respect to the relevant experimental data as is shown graphically in Figs 2–4. In these figures, points that fall on the 'main diagonal' line through the origin with a slope equal to one indicate perfect agreement between Nu_{cal} and Nu_{exp}. Sample sets of data, selected at random from each relevant data subset, are plotted in these figures because these sample data are sufficient for the present purpose and are not so numerous as to render their graphical representations confusing.

Equation (37) yields a maximum mean deviation of 6.2% for all relevant data subsets and a maximum error of -14.6% for any single datum. This level of accuracy overfulfills the criterion of acceptability, and renders equation (37) acceptable. Further, equation (37) overfulfills the criterion of acceptability in the transition regions (Darcy to Forchheimer and Forchheimer to turbulent) if the 'points of transition' approach described above ($Re_{DF} = 3$ and $Re_{FT} = 100$) is adopted.

CONCLUSION

It has been demonstrated that equation (37), with appropriate constants for Darcy, Forchheimer and turbulent flow, represents the entire body of data considered in this study with a degree of accuracy that is deemed acceptable for design purposes. This empirical correlation equation is based upon a hypothesis that regards the flow in a porous medium to be the superposition of a 'fine' component upon a 'coarse' component and takes into account the effect of the wall and the influence of dispersion upon heat transfer for the geometry considered herein. It is anticipated that the dimensionless measure of dispersion, Di, determined in the course of this study and defined by equation (36), will be applicable to forced convection heat (and mass) transfer for geometries other than the specific geometry considered here.

REFERENCES

1. R. M. Fand, B. Y. K. Kim, A. C. C. Lam and R. T. Phan, Resistance to the flow of fluids through simple and complex porous media whose matrices are composed of randomly packed spheres, *ASME J. Fluids Engng* **109,** 268–273 (1987).
2. L. H. S. Roblee, R. M. Baird and J. W. Tierney, Radial porosity variations in packed beds, *Am. Inst. Chem. Engrs J.* **4,** 460–464 (1958).
3. R. F. Benenati and C. B. Brosilow, Void fraction distribution in beds of spheres, *Am. Inst. Chem. Engrs J.* **8,** 359–361 (1962).
4. D. Mehta and M. C. Hawley, Wall effect in packed columns, *I & EC Process Design and Development* **8,** 280–282 (1969).
5. W. Reichelt, Zur berechnung des druckverlustes einphasig durchstroemter kuger-ung zylinder-schuettungen [Calculation of pressure drop in spherical and cylindrical packings for single phase flow], *Chemie-Ingenieur-Technik* **44**(18), 1068–1071 (1972).
6. R. M. Fand and R. Thinakaran, The influence of the wall on flow through pipes packed with spheres, *ASME J. Fluids Engng* **112,** 84–88 (1990).
7. R. M. Fand and K. K. Keswani, Combined natural and forced convection heat transfer from horizontal cylinders to water, *Int. J. Heat Mass Transfer* **16,** 1175–1191 (1973).
8. R. M. Fand and R. T. Phan, Combined forced and natural convection heat transfer from a horizontal cylinder embedded in a porous medium, *Int. J. Heat Mass Transfer* **30,** 1351–1358 (1987).
9. R. M. Fand, T. E. Steinberger and P. Cheng, Natural convection heat transfer from a horizontal cylinder embedded in a porous medium, *Int. J. Heat Mass Transfer* **29,** 119–133 (1986).
10. V. Prasad, N. Kladias, A. Bandyopadhaya and Q. Tian, Evaluation of correlations for stagnant thermal conductivity of liquid-saturated porous beds of spheres, *Int. J. Heat Mass Transfer* **32,** 1793–1796 (1989).
11. P. Cheng, Mixed convection about a horizontal cylinder and a sphere in a fluid saturated porous medium, *Int. J. Heat Mass Transfer* **28,** 1245–1247 (1982).
12. D. A. Nield and A. Bejan, *Convection in Porous Media.* Springer, New York (1992).
13. R. T. Phan, Combined forced and natural convection heat transfer from a horizontal cylinder embedded in a porous medium, M.S. Thesis in Mechanical Engineering, University of Hawaii (supervised by R. M. Fand) (1986).
14. G. C. Vliet and G. Leppert, Forced convection heat transfer from isothermal spheres, *J. Heat Transfer* **83,** 163–175 (1961).